Ing. Libor KOTAČKA

SECOND HARMONIC GENERATION IN OPTICAL WAVEGUIDES

GENERACE DRUHÉ HARMONICKÉ V OPTICKÝCH VLNOVODECH

PhD Thesis
PHYSICAL AND MATERIAL ENGINEERING

Supervisor: Prof. RNDr. Jiří Komrska, CSc.
Opponents: Prof. RNDr. Pavel Chmela, DrSc.
Doc. RNDr. Petr Malý, DrSc.
Dr. Ing. Ivan Richter

Date: October 11, 2001
Key Words
second harmonic generation, Čerenkov regime, nonlinear optical waveguides, phase matching

Klíčová slova
generace druhé harmonické, Čerenkovův režim, nelineární optické vlnovody, fázová synchronizace

Místo uložení
oddělení pro vědu a výzkum FSI VUT v Brně
# Table of Contents

Table of Contents.............................................................................................................................. 3  
1 Preface............................................................................................................................................. 5  
2 Introduction..................................................................................................................................... 7  
    2.1 The Goals of the Thesis .......................................................................................................... 7  
    2.2 Historical Survey....................................................................................................................... 8  
3 Methods and Approaches.............................................................................................................. 10  
    3.1 Čerenkov Regime SHG............................................................................................................ 10  
    3.2 BK Approach ............................................................................................................................ 14  
4 Conclusions; Essential Results of the Thesis .............................................................................. 19  
5 Shrnutí............................................................................................................................................... 21  
6 Author’s Publications Related to this Thesis.............................................................................. 24  
7 References......................................................................................................................................... 25  
8 Curriculum Vitae ............................................................................................................................ 27
1 PREFACE

It has already been nearly five years since I started my PhD study under the supervision of Prof. Jiří Komrska at the Institute of Physical Engineering, Faculty of Mechanical Engineering, Technical University of Brno. At the beginning when Prof. Komrska and I were mapping out the scope of my PhD study we were thinking along the lines of the optical diffraction on periodic structures, which I considered in my graduation project.

The extensive co-operation with the Biology department of the Medical Faculty of the Masaryk University in Brno (Prof. Janisch, Ing. Hřib) was devoted to the quasi-periodicity of the protein bodies in nigra seeds. Some results were reported at several, mainly electron microscopy meetings (see author’s publications).

While studying periodicity of biological samples another periodic structure caught my attention – the so-called Fibre Bragg Gratings (FBG). Despite having been extensively investigated since late 1980s FBGs were still relatively new subject at that time. This research was done in close cooperation with Ivo Procházka (TU Brno), and RNDr. Petráš (Palacký University Olomouc). Our research was enthusiastically supported by Prof. Chmela. Our joint effort resulted in one conference contribution and one paper on the general properties of the FBG published in 1999. There we examined the influence of the chirped and amplitude spatially modulated FBG on the reflection/transmission spectra.

After one and a half year, which I spent at TU Brno researching various diffraction phenomena as described above, Doc. Jiří Čtyroký from IREE AS CR invited me to join the Copernicus project INCO-COP 96 0194 (see below) concerning both the theoretical and technological aspects of the second harmonic generation in optical waveguides.

The main contribution to my PhD thesis has come from the research conducted at the University of Twente as a part of the project INCO-COP Compound Waveguide Structures for Efficient Frequency Doubling in Diode Pumped Short Wavelength Microlasers. There were four groups actively involved in the project: The University of Twente (UT) - Netherlands, CeramOptec GmbH - Germany, IOFAN-INFO Ltd. - Russia, and The Institute of Radio Engineering and Electronics (IREE) - Czech Republic. The aim of the project was to fabricate several different prototypes of intense (1-2 W) coherent light sources lasing at short wavelengths in the 600 nm to 400 nm range which in its final stage should be ready for commercial applications (ophthalmology). One of these prototypes was just under development in a close collaboration between the UT and the IREE. The essential part of the research activities was done at the University of Twente from April 1998 to September 1999.

Intensive experimental work required me to spend significant period of time in the laboratories and in the clean room facility at the UT. Since September 1999 the research work has continued at the IREE.
The research at the UT encompassed not only the theoretical study of the entire field including computer simulations, but also the design, fabrication and, finally, the testing of all the prototypes. The whole project became more complex than it had been originally expected. The time spent then at the IREE was except of completing measurements started at the UT devoted mainly to the theoretical study of the SHG phenomena.

This PhD thesis is intended to describe the essential results obtained during my postgraduate study. The thesis is divided into two parts. The first part briefly summarises the PhD study and the attained results. New theoretical facts discovered during my study are, however, described in details. The second part contains all the publications and contributions on the second harmonic phenomena in optical waveguides which have already been published or which were recently accepted for publication.

A large number of persons have contributed, either directly or indirectly, to this thesis. I am indebted to Prof. RNDr. Jiří Komrška, CSc, my principal supervisor, for excellent guidance throughout the whole of my PhD studies and for very enjoyable support during the last ten years. I am also thankful to Doc. Ing. Jiří Čtyroký, DrSc., whose driving initiative played an important role in the collaboration between the IREE and the UT and during countless discussions on the project planning and problem solutions. His role in my still developing scientific carrier has never been more crucial then now. I am indebted to Dr. Ir. Hugo J. W. M. Hoekstra, the coordinator of the INCO-COP project, who has in the course of time become a partial supervisor of my PhD studies. Although I left the University of Twente nearly two years ago, our close co-operation still fruitfully continues. Thanks are also due to Prof. RNDr. Pavel Chmela, DrSc. for kindly discussing the overall non-linear optics and the optical properties of anisotropic media in particular.

The support of large number of individuals at TU Brno, MESA+, LDG, IREE, and some other institutions is gratefully acknowledged as well. My particular thanks go to Prof. RNDr. František Melkes, CSc., Doc. RNDr. Bohumila Lencová, CSc., Prof. Evgenij A. Lapšin, DrSc., Prof. RNDr. Michal Lenc, PhD., Fabien Bayle, MSc., Ing. Jiří Janta, CSc., Ing. Justin Asma, and Bc. Mathijs Weenk for kindly discussing the mathematical, computational, and optical aspects. Thanks are also due to Ing. Jiří Sláma for assistance with writing the C+ code necessary for the numerical simulation of the SHG phenomena, to Drs. Toon Adringa, Mr. Henk A. G. M. van Wolferen, Ir. Wichert Kuipers, and Drs. Sami Musa for the technical assistance in a laboratory during my stay at University of Twente, and to Ildar F. Salakhutdinov, PhD. for his co-operation on the ARM structure. The work of Mr. František Ondráček, Mr. Milan Hubálek, and Mr. Václav Drahoš on the various characterisations of the optical waveguides and on the final testing of the ARM structure is gratefully acknowledged. I am particularly indebted to Mrs. Alena Bidlová thanks to her amazing librarian skills I got all the papers I needed. Finally,
Ivo Procházka is acknowledged for his previous co-operation on FBG, and for his assistance with the English edition of this discourse.

I would like to dedicate this thesis to my best friends, Justin, Mathijs, Jenny, Theun, Frouke, Robert, Arjan, and Nicole, who lived with me in Parameceum during my stay in Holland. I am forever grateful for their support and friendship. And also to my dear friend Bob who encouraged me in the writing of my thesis from as far as California. Finally, to Yvona for her loyalty and something more than friendship.

Saint Etienne, May 2001

Libor Kotačka

2 INTRODUCTION

2.1 The Goals of the Thesis

The guiding properties of electromagnetic waves were for the first time reported more than one hundred years ago when Lord Rayleigh [1] presented his treatment on infinitely long dielectric tubes (cylinders) of an arbitrary cross-section area and bounded by perfect conductor. The most important sections, circular and rectangular ones, were studied in detail there. The boom in optical waveguides came sixty years later since the technology necessary for production of optical waveguides was simply not available until 1950s.

The arrival of optical waveguides coincided with the appearance of the SHG. The first work which dealt with the optical second harmonic generation (SHG) phenomena dates back to 1960s (for details please see the historical overview given below).

Both optical waveguides and the SHG phenomena entered the research community at the same time and one can, therefore, say that they have been continuously studied for last forty years. The need for the high pump radiation density for an efficient SHG effect has been well-known since the first theoretical work concerning this phenomenon. The advantages of diffraction free and well
constrained guided waves for SHG is obvious (see also Yariv’s remark cited as [5] in the following theoretical Chap. 3)

Compact coherent light sources lasing at wavelengths from 600 nm down to green or blue light has been researched since 1970s. The SHG has played relatively important role beside other techniques due to a lack of available materials (note however the recent appearance of GaN structures). Recent proliferation of different applications, particularly in contemporary optics, demanding coherent blue or green light has stimulated the research even further. The INCO-COP project is a good example of that.

There are several methods how to efficiently generate coherent green or blue light. Their description is, however, for the sake of brevity omitted (for more information please refer to Discourse of the PhD thesis, March 2000). The SHG in optical waveguides still seems to be interesting for the green/blue light generation. The SHG plays important role not only in the light generation but also, for example, in the SHG microscopy, in characterisation of various non-linear waveguides (even those including gratings) etc. Finally, as will be shown below, the so-called Čerenkov regime may offer relatively cost-effective way to achieve efficient generation without the usual requirements on the phase matching.

The goal of my PhD research was to design and fabricate a coherent green-blue light source. The research was supposed to follow up on previous investigations done at The University of Twente. The prototype device was to be made in the clean room facility available there. The SHG based lasing source research was later extended to abnormal reflecting mirror (ARM) which is the current aim of the further co-operation between the UT and IOFAN-INFO in a framework of the NWO project. My results were taken as the base for the further research.

As to the theoretical part, the topics which are presented in this thesis appeared successively during my study at the UT and the IREE. At the beginning of my work at Twente, we recognised that a comprehensive study of the Čerenkov regime SHG (ČSHG) had to be done. We soon noticed that the transition point between the ČSHG and the classical guided-guided interaction (GSHG) had been given only a slight attention. Hence, Doc. Čtyroký, Dr. Hoekstra, and I invested a lot of effort into the detail understanding of the SHG phenomena. Although it is still being researched, some of our results are believed to have important impact on the overall understanding of the SHG phenomena (see Chap. 3).

### 2.2 Historical Survey

The observation of the SHG in optical waveguides followed the first SHG experiments in bulk crystals in early 1960s. Note that the non-linear optics was born at that time as well. Although the SHG in Čerenkov regime was reported by Tien et
for the first time in 1970, most theoretical and experimental papers on this topic appeared only in 1990s.

In the following the history of evolution of the SHG in optical waveguides is briefly reviewed with the emphasis placed on essential events, experiments, and theoretical papers. All the papers important not only for the development of the SHG in waveguides in general but also for the PhD thesis itself are listed. References are, for the sake of brevity, omitted unless directly related to this thesis (see Chap. 3).

1961 - the beginning of the nonlinear optics, Franken et al.; SHG in quartz crystal
1962 - the harmonic generation and wave mixing in a layered configuration considered by Bloembergen and Pershan, and appearance of pioneering papers (Armstrong et al., Kleinman)
1968 - Smith; PM in four-layered optical (GaAs) waveguides, Boyd and Kleinman published the "BK theory"
1970 - Tien et al.; the first observation of the SHG in Čerenkov regime
1973 - Yariv; the coupled mode theory of the guided optics
1974 - Burns observed non-critical PM SHG
“1980s” - LiNbO₃ Ti-indifused channel waveguides
“late 1980s” – appearance of the periodical domain inversion; QPM techniques
1991 - Tamada presented the CMA study of the ČSHG (this theory caused infinitely peaked efficiency)
1992 - Asai et al. reported the infinite peak for the first time; Fluck - 29 % total conversion efficiency (in waveguides)
1993 - Shenoy et al. analysed the waveguide length dependence (losses), and intensity profiles of the Čerenkov radiation
1994 - Doumuki et al. reported approximately 14 % total conversion efficiency in the ČSHG and experimentally demonstrated the transition between the peak in the Čerenkov regime and the classical phase-matching (KTP/Ta₂O₅/SiO₂)
1998 - Chang and Shaw theoretically analysed the origin of the infinite peak in the conversion efficiency in the Čerenkov regime (MNA as the nonlinear material)
1999 - Fluck extended the BK theory to waveguides
2000 - Čtyroký and Kotačka showed (the CK theory) that the conversion efficiency for small Čerenkov angles at the vicinity of phase matching obeys approximately the law $\eta_{2\omega} \propto L^{3/2}$
2001 --- Kotačka et al.; the SHG efficiency may be generally expressed (based on the BK theory) as $\eta_{2\omega} \propto L^p$, $p \in \{1, 2\}$ with respect to any “leakage” of the generated radiation. Further, they found the exact solution which describes the SHG phenomena in various circumstances.
To highlight the focus of this PhD thesis, we will give an overview of the history of the ČSHG evolution. Tamada presented the ČSHG theory in 1991. The theory, however, diverged for very small Čerenkov angles. This was reported and approximately explained by Asai et al. one year later. Doumuki et al. experimentally showed in 1994 that the maximum of the conversion efficiency in the ČSHG takes place in the transition point between the two possible regimes (i.e. between the ČSHG and the GSHG). In 1998, Chang and Shaw explained the origin of this point. 1998 was also the year when I began my research. The results are summarised in the conclusion.

3 METHODS AND APPROACHES

The goal of this chapter is to briefly summarise theoretical methods and approaches to explain the second harmonic phenomena in planar optical waveguides. This mathematical account contains most of the new theoretical results concerning the SHG in waveguides as were obtained during my PhD study.

3.1 Čerenkov Regime SHG

This section describes basic properties of the Čerenkov regime SHG (ČSHG). Only relevant theoretical expressions are presented and adapted. Furthermore, only the $TE^\omega - TE^{2\omega}$ conversion is considered because the results for the $TM$ modes may be derived in a similar fashion as for $TE$ modes. The conversion of the fundamental $TE^\omega$ guided mode to the second harmonic $TE^{2\omega}$ radiation mode is analysed using the coupled mode analysis applied to a three-layer slab optical waveguide.

![Fig. 1: The phase-matching diagram in the case of ČSHG.](image)

A waveguiding high-refractive index layer of the thickness $h$ made from an optically linear material is deposited on top of a non-linear substrate (in our configuration only the substrate is a non-linear medium) and covered by a lower-index superstrate. The $z$ axis determines the propagation direction of the fundamental guided mode. The principle of the ČSHG is depicted in Fig. 1, i.e. the phase matching between the pump guided mode and the generated second harmonic
mode is satisfied automatically [2]. The second harmonic radiation leaks into the substrate under the Čerenkov angle and obeys the relation
\[
\cos \theta = \frac{N_\omega}{n_{s,2\omega}}.
\] (1)

\(N_\omega\) is the effective index of the fundamental guided mode and \(n_{s,2\omega}\) is the refractive index of the substrate for the second harmonic radiation.

If we follow the procedure described by Tamada [2] then the distribution of the guided fundamental \((\omega)\) field in the substrate (i.e. for \(x < -h\)) obeys the relation
\[
E_{y,\omega} = A_g \left[ \cos(\kappa h) + \frac{\delta}{\kappa} \sin(\kappa h) \right] \exp[\gamma(x + h)],
\] (2)

where \(\kappa = k \left(n_{g,\omega}^2 - N_\omega^2\right)^{1/2}\), \(\gamma = k \left(N_\omega^2 - n_{s,\omega}^2\right)^{1/2}\), \(\delta = k \left(N_\omega^2 - n_{c,\omega}^2\right)^{1/2}\), with \(k = 2\pi / \lambda\). The subscripts \(g, s, c\) relate to the guide, substrate, and cladding, respectively. The normalisation constant \(A_g\) is given by (for detail theoretical description please refer to [3, Chap. 1])
\[
A_g = \left(\frac{4\omega \mu_0 \kappa^2}{\beta_\omega (\kappa^2 + \delta^2)(h + \gamma^{-1} + \delta^{-1})}\right)^{1/2}
\] (3)
as a direct consequence of the condition \(\int_\infty^{-}\int_\infty \left|E_{y,\omega}\right|^2 dx = 1 \text{ [W/m]}.\) The second harmonic field is expressed as a radiation mode
\[
E_{y,2\omega} = A_r \times
\left\{ \left[ \cos(\sigma x) + \frac{\Delta}{\sigma} \sin(\sigma x) \right] \cos[\rho(x + h)] + \frac{\sigma}{\rho} \left[ \sin(\sigma x) - \frac{\Delta}{\sigma} \sin(\sigma x) \right] \sin[\rho(x + h)] \right\}
\] (4)

where \(\sigma = 2k \left(n_{g,2\omega}^2 - N_\omega^2\right)^{1/2}\), \(\rho = 2k \left(n_{s,2\omega}^2 - N_\omega^2\right)^{1/2}\), \(\Delta = 2k \left(N_\omega^2 - n_{c,2\omega}^2\right)^{1/2}\), with \(k = 2\pi / \lambda\). The normalisation constant \(A_r\) is given as follows
\[
A_r = \left(\frac{8\omega \mu_0 \sigma^2 \rho^2}{\pi \beta_\omega \left[ \sigma \sin(\sigma h) - \Delta \cos(\sigma h) \right]^2 + \rho^2 \left[ \sigma \cos(\sigma h) + \Delta \sin(\sigma h) \right]^2} \right)^{1/2}.
\] (5)

Next, the wave equation with a perturbed polarisation vector is to be solved (for details please see e.g. [2] or [4]). The derivation is for the sake of brevity omitted here.

The solution to the wave equation leads to the following expression describing the generated second harmonic power \(P_{2\omega}\) as the function of the pump power \(P_\omega\) and the interaction length \(L\) (we integrate over all radiation modes because of the continuous spectrum of the radiation mode propagation constant)
\[
P_{2\omega} = P_\omega L^2 \int_0^\infty \eta \rho_{pm} \ d\rho.
\] (6)
The quantity 2 is the so-called normalised conversion efficiency given by
\[ \eta = d_{33}^2 \omega^2 \varepsilon_0^2 |F|^2, \]  
(7)

where \( \omega \) is the frequency of the fundamental radiation, \( \varepsilon_0 \) is the permittivity of the vacuum, and \( d_{33} \) is the nonlinear coefficient (\( d_{33} \) plays an essential role in the configuration exploiting the KTP for the \( TE^\omega - TE^{2\omega} \) conversion). The overlap integral \( F \) is defined by

\[ F = \int_{sub} E^2_{y,\omega} E_{y,2\omega} dx. \]  
(8)

Substituting Eqs. (2) and (4) to Eq. (8), the overlap integral yields

\[ F = A_g^2 A_r A^2 \int_{-\infty}^{h} \exp[2\gamma(x + h)] \left\{ B \cos[\rho(x + h)] + C \frac{\sigma}{\rho} \sin[\rho(x + h)] \right\} dx, \]  
(9)

with \( A = \cos(\kappa h) + (\delta / \kappa) \sin(\kappa h) \), \( B = \cos(\sigma h) + (\Delta / \sigma) \sin(\sigma h) \), and finally \( C = \sin(\sigma h) - (\Delta / \sigma) \cos(\sigma h) \). A simple integration then gives

\[ F = A_g^2 A_r A^2 \left[ \frac{2B\gamma - C\sigma}{4\gamma^2 - \rho^2} \right]. \]  
(10)

The term \( I_{pm} = \sin^2(\Delta_{pm} L / 2) / (\Delta_{pm} L / 2)^2 \), with \( \Delta_{pm} = 2\beta_{2\omega} - \beta_{2\omega} \), describes the phase mismatch between all possible radiation modes and the guided pump mode. The overlap integral is slowly varying function of \( \rho \) compared to the mismatch factor \( I_{pm} \) and can be, therefore, left out the integration sign. If we consider relatively large Čerenkov angles (say \( \theta > 2^\circ \)) and also relatively long propagation length \( L \), the integral in Eq. (6) may be approximately expressed as (with the help of \( \rho^2 = 4n_{s,2\omega}^2 k^2 - \beta_{2\omega}^2 \rightarrow d\rho = d\Delta_{pm} \beta_{2\omega} / \rho \))

\[ \int_{-\infty}^{h} I_{pm} d\rho = \beta_{2\omega} \rho^{-1} \int_{-\infty}^{h} I_{pm} d\Delta_{pm} = 2\pi\beta_{2\omega} / (L\rho). \]  

Following Fig. 1 we may write

\[ \cot \theta = \beta_{2\omega} / \rho. \]  

Hence,

\[ P_{2\omega,C} = 2\pi\eta LP_{2\omega}^2 \cot \theta. \]  
(11)

Note that the generated second harmonic power is proportional to the propagation length only for large Čerenkov angles. The relation (11), however, diverges for small Čerenkov angles.

Assuming small Čerenkov angles, i.e. \( \rho << 1 \), we may approximately express the mismatch factor as

\[ \Delta_{pm} = 2\beta_{\omega} - \sqrt{4k^2 n_{s,2\omega}^2 - \rho^2} \approx 2\beta_{\omega} - 2kn_{s,2\omega} + \frac{\rho^2}{4kn_{s,2\omega}}. \]

The integral in (6) then takes the form (we may again extend the integration interval, because of sharply peaked behaviour of the “sinc” function)

\[ \int_{-\infty}^{\infty} \sin^2 \left[ \frac{(2\beta_{\omega} - 2kn_{s,2\omega} + \rho^2 / 4kn_{s,2\omega}) L / 2}{2} \right] d\rho \approx \frac{16}{3} \frac{2\pi kn_{s,2\omega}}{\sqrt{L}}. \]
We considered just the vicinity of the well phase-matched interaction, i.e. \( \beta_{\omega} \simeq kn_{s,2\omega} \). Substituting this result back into Eq. (6) we get the relation for the ČSHG for small Čerenkov angles close to the phase-matched interaction

\[
P_{2\omega,\text{peak}} = \frac{8}{3} \sqrt{2\pi kn_{s,2\omega}} \eta b_{\omega}^2 L^{3/2}.
\]

The subscript \( \text{peak} \), as it will be seen later, refers to the fact that the validity of this relation is limited only to the peaked Čerenkov conversion efficiency (see Fig. 2), where the generated SH power follows \( P_{2\omega} \propto L^{3/2} \). This result was first reported in [4] and may be understood as the transition between the ČSHG and the classical guided-guided SHG interaction described e.g. in [5] where the generated SH power exhibits the well-known quadratic dependence on the propagation length.

Let us further elaborate on the influence of the overlap integral on the conversion efficiency behaviour for small Čerenkov angles because the maximum of the conversion efficiency is expected to be in this region. As was said, if \( \rho \to 0 \) for \( \theta \to 0^\circ \) then it can be seen from the normalisation constant of the radiation SH mode given in (5) that this constant is for \( \rho = 0 \) equal to zero everywhere except of the very close vicinity given by

\[
\sigma \sin(\sigma h) - \Delta \cos(\sigma h) = 0,
\]

which yields the modified dispersion relation giving the efficiency (see also [6])

\[
\frac{\arctan(\delta / \kappa) + \arctan(\gamma / \kappa)}{\kappa} = \frac{\arctan(\Delta / \sigma) + \pi}{\sigma} = 0.
\]

Solving the modified dispersion relation (14) one can find that there is only one suitable wavelength, which can be converted with the highest conversion efficiency for a given refractive index. Moreover, this can be achieved only with a particular thickness of the guiding layer given by

\[
h = \frac{[\arctan(\delta / \kappa) + \arctan(\gamma / \kappa)]}{\kappa}.
\]

Similar expression can be found for four-layer systems, which is the matter of our study (a grating may be understood as an extra layer of specific properties).

Finally, the peak exhibits rather narrow FWHM (see Fig. 2) similar to the guided-guided SHG behaviour. The peak is the first point when the pure phase matched SHG occurs. This creates hurdles in a device fabrication (e.g., an accuracy of the guide thickness +/- 0.2 nm is required, similar remark holds for the pump wavelength). Despite such obstacles we view it as a promising way forward to exploit the Čerenkov regime (apart from its peaked conversion efficiency) because the phase matching is automatically satisfied.

The sharply peaked second harmonic efficiency in the transition point was experimentally studied in two papers by Doumuki \textit{et al.} [7, 8]. We recognised in [4, Fig. 7b]) and [6, Eq. (7)] that the generated SH power in the peaked ČSHG conversion efficiency approximately obeys (please note the difference of the factor of 2 missed in [4, 6]) the relation
\[ P_{2\omega,\text{peak}} \approx 0.12 \left( \frac{L}{L_{1\text{mm}}} \right)^{3/2} P_{\omega}^2, \]  

(15)

where the bracketed term denotes the relative interaction length related to 1 mm. The pump power in the relation (15) is understood to be normalised to 1 \( \mu \)m of the slab width.

\[ \text{Fig. 2: The conversion efficiency of 1 mm long KTP/Si}_3\text{N}_4/\text{SiO}_2 \text{ device calculated by Eqs. (11)} \text{ and (12), respectively (Eff. } = P_{2\omega}/P_{\omega}^2 \text{ [\mu m/W]). The ČSHG does not occur in the dark area in the right edge of the graph as it is the GSHG domain.} \]

### 3.2 BK Approach

Our theory introduced in [4] (in the following referred to as the CK theory) is discrete with respect to the exponent of the propagation length \( L \). Thus, we shall now offer an alternative approach to the CK theory.

Fluck [9] extended the Boyd and Kleinman theory [10] (the BK theory) to non-linear interactions of SHG beams of Gaussian profiles in waveguides. Both Fluck and Boyd-Kleinman works considered a “walking-off” of the second harmonic light under exactly (depending by material properties) defined angle. The radiated SH power does not necessarily take the form of the Gaussian beam. In the following, we will replace this walk-off angle by the Čerenkov one \( \theta \) as defined in the previous paragraph. We will then follow the approach [9, 10] because of certain similarities between the Čerenkov regime and the interaction of guided waves as a “standard” SHG process with varying walk-off angle.
The BK theory defines three variables
\[ \sigma = \Delta k b / 2, \quad \beta = \theta b / 2 w_0, \quad \xi = L / b. \] (16)

For Gaussian pump beam, \( b = k_\omega w_0^2 \) is the confocal parameter, and \( w_0 \) is the Gaussian beam radius. \( \Delta k = 2k_\omega - k_{2\omega} \) is the mismatch between the fundamental and second harmonic wave vectors.

The general expression for the generated SH power in the planar waveguide takes the form [9]
\[ P_{2\omega} = \eta P_\omega^2 L^{3/2} g(\sigma, \beta, \xi), \quad \text{with} \quad \eta = \frac{8\pi^2}{\varepsilon_0 c \lambda^2} \frac{d_{eff}^2}{N_{2\omega} N_{\omega}^2} \sqrt{2 N_{\omega} / \lambda} |F|^2. \] (17)

\( \varepsilon_0, c, \lambda, d_{eff} \) are the vacuum permittivity, velocity of light in vacuum, the wavelength of the pump radiation, and the effective non-linear constant, respectively. \( N_{\omega}, N_{2\omega} \) represent the effective refractive indices of the pump and the SH radiation, respectively. Finally, \( F \) is the mode field overlap integral defined in (8). Both the fundamental and second harmonic fields in the integral are expected to be normalised to the unit transmitted power as shown in the previous paragraph. Note that only the substrate is again assumed to exhibit non-linear properties.

\( g(\sigma, \beta, \xi) \), a function of variables that are to be optimised for high conversion efficiency, describes the conversion efficiency behaviour for all SHG regimes, i.e. not only for the ČSHG \( (\sigma = 0) \) and GSHG \( (\beta = 0) \) regimes but also for the transition region between them. Following [9, 11] this function may be in the case of “weak focusing” (i.e. for \( \xi < 1 \)) expressed in the form
\[ g(\sigma, \beta, \xi) = 2 \sqrt{\frac{\xi}{\pi}} \int_{-\infty}^{\infty} \sin^2 \left( \frac{\sigma \xi + 4 \beta \xi s}{4(\sigma \xi + 4 \beta \xi s)^2} \right) \exp(-4s^2) ds. \] (18)

This integral is complicated enough to be expressed analytically. The ČSHG case was studied in [11] by exploiting the hypergeometric series.

Comparing the first two terms of the Taylor’s series we get
\[ \frac{\sin^2(bx)}{(bx)^2} \approx \exp \left( -\frac{b^2}{3} x^2 \right). \] (19)

The integral in (18) then takes the form
\[ \int_{-\infty}^{\infty} \left[ -\frac{b^2}{3} \left( \frac{a}{b} + s \right)^2 \right] \exp(-4s^2) ds = \frac{\sqrt{3\pi}}{\sqrt{b^2 + 12}} \exp \left( -\frac{4a^2}{b^2 + 12} \right), \]
where \( a = \sigma \xi \) and \( b = 4\beta \xi \). Hence, we obtain the desired expression
\[ g(\sigma, \beta, \xi) \approx \frac{\sqrt{\xi}}{\sqrt{1 + 4\beta^2 \xi^2 / 3}} \exp \left( -\frac{\sigma^2 \xi^2}{4\beta^2 \xi^2 + 3} \right). \] (20)

This may be finally rewritten, with respect to the relation (17), into the form
\[ P_{2\omega} \propto \frac{L^2}{\sqrt{1 + 4\beta^2\xi^2 / 3}} \exp\left( -\frac{\sigma^2 \xi^2}{4\beta^2\xi^2 + 3} \right). \]  

where \( \sigma \xi = \Delta k L / 2 \). In the following we engage in the analyses of Eq. (21).

Let us first study both limiting cases. To do so we must introduce an extra normalisation \( \xi = 1 \) when \( L \) reaches its maximal value in order to avoid ambiguous results. Eq. (21) for the GSHG \( (\beta = 0) \) yields (compare [5])

\[ P_{2\omega,G} \propto L^2 \exp\left( -\frac{\sigma^2 \xi^2}{3} \right) = L^2 \frac{\sin^2 (\sigma \xi)}{(\sigma \xi)^2}. \]  

Taking advantage of the fact that \( \sigma = 0 \) in the case of automatically satisfied phase-matching in the ČSHG regime, Eq. (20) yields

\[ g(\beta, \xi) \equiv \frac{\sqrt{\xi}}{\sqrt{1 + 4\beta^2\xi^2 / 3}}. \]  

If we for example set \( \theta = 15^\circ \) then \( \beta \equiv 5 \) and we can write

\[ P_{2\omega,C} \propto L / \theta. \]  

This results agrees with our assumptions in Eq. (11) and also [2, 4].

In the next step we shall analyse the first term in (21), i.e. the one which does not depend on the phase matching condition to be satisfied. We can see from Fig. 3 that the generated SH power exhibits for \( \theta = 0^\circ \) (i.e. the GSHG limit) the quadratic dependence on the propagation length (for large Čerenkov angles the dependence is linear).

\[ \text{Fig. 3: 3D plot of the first term in Eq. (21).} \]

This fact offers an opportunity to rewrite the dependence of the first term in (21) as the exponent of \( L \). If we assume that the exponent of the propagation length
continuously decreases with the increasing Čerenkov angle, we get the general dependence for the SH power with the Čerenkov angle as

\[ P_{2\omega} \propto L^{p}, \text{ where } p(\theta) \in (1,2). \tag{23} \]

To prove the validity of just presented theory based on the assumptions (23), we have fitted the data obtained from the first term of the expression (21) and compared them [11] with the data yielded by the beam propagation method (BPM; the BPM calculations were performed by Dr. Hoekstra). The exponent of \( L \) was calculated by the least-square method for each case.

![Exponent of the length depending on the "leaking" angle](image)

**Fig. 4:** The fitted values of the interaction length exponent as the function of the Čerenkov (or leaking) angle (\( L=0.14 \) mm). Solid line – BPM data, dashed line – presented modified BK theory (phase-matching factor in Eq. (21) is omitted).

Fig. 4 shows that for \( \theta > 2^\circ \) both mentioned methods yield similar results. The difference occurring for \( \theta < 2^\circ \) is not well understood yet. The non-quadratic dependence at the beginning of the GSHG curve (\( \theta = 0^\circ \)) may be explained by the cut-off vicinity of the guided SH mode where some substrate modes can still be excited – the SH radiation may be slightly leaking out of the waveguide. Although we have neglected the influence of the phase matching factor in Fig. 4, the observed deviation between the data is not probably caused by this. Moreover, the phase-matching in the vicinity of the transition point is difficult to exactly define because of the continuum spectrum of the radiation modes propagation constant.

The general behaviour of Eq. (21) can be seen from Fig. 5. The study of this effect is still underway.
It can be further shown (see the poster joined to [11]) that the shorter \( L \) is the sooner the exponent of \( L \) increases with respect to the Čerenkov angle and *vice versa*. This phenomenon is of general nature. The very recent results indicate that the SHG conversion efficiency is linear or quasi-linear even for relatively small Čerenkov angles when \( L > 1 \) mm and then steeply increases to the quadratic dependence in the case of the GSHG. At the transition point between the CSHG and the GSHG regimes it takes a value from \( p = 1.7 \) to \( p = 1.6 \) for relatively short and relatively long interaction lengths (longer or shorter than 0.5 mm), respectively. Further compare the approximate value \( p = 1.5 \) reported in [4], which seems to be valid only in the peak for any interaction length.

Comparing our results with those of Fluck [9] we conclude that *any leakage* of the SH radiation out of the pumped region (e.g. Čerenkov regime [2, 4], “walk-off” due to the double refraction [10], or simply the beam divergence of the generated light in planar waveguides [9]) effectively decreases the exponent of the \( L \)-dependence of the SH power from its maximum value 2 to 1.

![Fig. 5: 3D plot generated SH power in Eq. (21) as the function of the propagation length and the parameter \( \beta \) (describing actually any leakage of the SH radiation). Note the crucial role of the phase matching factor, i.e. the maximum conversion does not occur for \( \beta = 0^\circ \) as could be expected.](image)
4 CONCLUSIONS; ESSENTIAL RESULTS OF THE THESIS

I will conclude my PhD thesis with a survey of all the contributions and papers which are pertinent to this thesis and which I have co-authored. The full list of all contributions is given below.

The first contribution [1] was reported after preliminary research at the beginning of my stay at Twente. In this paper we proved by the means of two independent methods the existence of a sharp peak in the conversion efficiency in the Čerenkov regime. We also reported that commonly used theory yielded the infinite value of the peak. Two other posters [2, 3] immediately followed this contribution. Poster [2] dealt with the transition between Čerenkov and phase-matching regimes and presented the general expression for the SHG conversion (it has to be, however, solved numerically). The limiting cases for large and small Čerenkov angles were discussed separately and the dependence $L^{3/2}$ of the SHG efficiency close to the phase-matched transition point was reported for the first time ever. The poster [3] introduced the condition for the peak position in the “wavelength-thickness diagram”. The four-layer structure was discussed in order to optimize the ČSHG peaked efficiency. Finally, strict demands on material applied and technological processes were pointed out.

Most of the results from [1, 2, 3] were summarized and officially reported in [4] (the CK theory, see also Chap. 3). The theory describing the TE-TE conversion was rigorously derived and some theoretical aspects of it were discussed. The paper concluded that there were three regimes obeying the linear, quadratic, and $L^{3/2}$ dependence, respectively. The following poster [5] extended the CK theory to the more complex TM-TM case. Among other advantages the approximately eight times higher efficiency at the peak position for a similar waveguide arrangement (i.e. exploiting the same non-linear coefficient as in [4]) was reported.

Other ways to reach the efficiency peak were proposed in the oral contribution [6], which actually followed up on the results reported in the poster [3]. The summarizing lecture [7] was devoted to the essential results obtained in the Copernicus project. The lecture comprised the complete theoretical, technological, and experimental description of the ARM specimen.

The paper [8] studied in detail the impact on the efficiency peak position when the demands on the technology process are less restrictive. Both widely available materials and a fictitious one were considered. The paper was based on [3, 6].

The recent progress, in comparison to the contribution [7], was reported in the presentation [9]. The poster [10] and the oral contribution [11] summarised the
research currently done on the modified BK theory (please see also the second part of Chap. 3).

The extended version of the lecture [7] was recently accepted for publication [12]. Finally, the detailed study of the TM-TM interaction based on the CK theory [4] was accepted as well [13]. This paper extends the poster [5] and presents along with [4] detailed theoretical treatment of the SHG in planar optical waveguides with the non-linear substrate.

To summarise my PhD research I would like to outline its main results. As to the theory, it was shown that the commonly used approach describing the SHG in optical waveguides is valid only in very specific circumstances. The detailed study [4, 13] showed that the SH power obeys in the vicinity of the phase-matching $L^{3/2}$ dependence close to the cut-off of the guided SH mode. Further, the conversion efficiency is reasonably high for the TM configuration. Some less restrictive ways to reach the conversion maximum utilising the four-layer system were shown in [8] – the results promise future advances as far as the technology is concerned. The ARM structure made on the non-linear waveguide generated in the Čerenkov regime about 0.3 $\mu$W under 2 W pumping (please note the very short interaction length) [12].

I spent most part of my stint at Twente trying to fabricate such a device. The final theoretical part of my PhD study [10, 11], devoted to the non-linear interaction of Gaussian beams in nonlinear media, appears to be the perhaps most challenging aspect of the whole research. The modified BK theory can be applied when describing the second harmonic interactions with the radiated power leaking out the device. The SH power dependence, with respect to the angle of the leakage, continuously changes the exponent of the propagation length – from the quadratic dependence in the case of the GSHG to the linear one in the case of the large angle ČSHG.
5 SHRNUTÍ

Předložená disertační práce se zabývá studiem generace druhé harmonické (GDH) v optických nelineárních vlnovodech. Většina výsledků uvedených v této práci vznikla v době mého ročního studijního pobytu v Nizozemí (University of Twente) a následného pobytu na ÚRE AV ČR, kde jsem byl zaměstnán v loňském roce. Vzhledem k charakteru disertační práce (soubor uveřejněných prací [1-13]; viz následující kapitola) je shrnutí podáno jako stručný komentář jednotlivých prací.


Spojením tzv. „ARM“ (abnormálně odrážející zrcadlo) struktury s nelineárním vlnovodem se podařilo vygenerovat (ČGDH režim) přibližně 0.3 $\mu$W při 2 W čerpacího výkonu (važte, prosím, velmi krátkou interakční délku při hodnocení souhrnné účinnosti) [12]. Zde bych měl podotknout, že většinu času jsem v Nizozemí věnoval technologii v „čistých prostorech“, což se již dnes ukazuje pro budoucnost jako nedocenitelná zkušenost.

Závěrečná část mého doktorandského studia [10, 11], věnovaná studií velmi obecné BK teorie interakce světla v nelineárním prostředí, se zdá být možná nejpřínosnější výsledek celého studia. Tato modifikovaná BK teorie efektivně popisuje (alespoň aproximativně) jakoukoli GDH interakci i v případech, kdy generované záření nějakým způsobem uniká ze struktury (resp. ze směru určovaného směrem šíření čerpacího záření). Generovaný výkon tak plynně mění závislost na interakční délce od kvadratické (VGDH) po lineární (ČGDH pro relativně velké úhly) s ohledem na rostoucí úhel.
6 AUTHOR’S PUBLICATIONS RELATED TO THIS THESIS

(The ratios behind determine the percentage contributions of the authors.)


7 REFERENCES


8 CURRICULUM VITAE

Personal Details:
Name: Libor Kotačka
Date and Place of Birth: 11th April 1973 in Velké Meziříčí, Czech Republic
Address: Jihlavská 249, Velká Bíteš 595 01, Czech Republic
Status: Single
Nationality: Czech

Education:
1996-present PhD study at The Brno University of Technical, Faculty of Mechanical Engineering, Institute of Physical Engineering. In my PhD thesis I considered the general study of SHG in optical waveguides, especially the so-called Čerenkov regime. The thesis concluded the three year long research in the non-linear optics.
In the beginning, I studied general diffraction properties of non-periodic structures. The results were immediately applied to the determination of the degree of quasi-periodicity of biological specimens (protein bodies inside the pine seeds). At the same time I investigated general properties of spatially modulated fibre Bragg gratings

Since September 2000 to present University Jean Monnet, Saint Etienne, France
My task was to develop a precise displacement sensing read-head for rotational bodies. The project was done in the co-operation between Jean Monnet University and SNFA Valenciennes.

Since April 1998 to August 2000 IREE AS CR, Prague, Czech Republic
I participated in the Copernicus project focused on the SHG phenomena and done in co-operation between The University of Twente and the IREE. Apart from waveguides characterisation I studied fundamental aspects of the SHG phenomena.
April 1998 to August 1999  University of Twente, The Netherlands
I collaborated on a project whose goal was to develop green coherent light source based on the second harmonic phenomena. I also spent a considerable amount of time in the clean room facility fabricating prototype devices. The project resulted into the connection of the SHG in the Čerenkov regime together with the abnormal reflecting mirror.

1991–1996  The Technical University Brno, Faculty of Mechanical Engineering
As my final project I considered the analysis of non-periodic images by the influence of the spatial frequencies.

1987–1991  Gymnázium, Velké Meziříčí, Czech Republic

**Interests and Activities:**
The radio-amateur (short waves), classical electronics, acoustics-theory of sound.

**Other Skills:**
I am fluent in both written and spoken English and Russian, now beginning French.
I am skilled in working with PC.
I have possessed a driving license since 1991.

Brno, May 2001
Libor Kotačka