Modeling of the Strange Behavior of the Selected Nonlinear Dynamical Systems. Part I: Oscillators
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MODELOVÁNÍ ZVLÁŠTNÍCH JEVŮ VE VYBRANÝCH NELINEÁRNÍCH DYNAMICKÝCH SYSTÉMECH. ČÁST I: OSCILÁTORY

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ABSTRACT

This part of the monograph is focused on the design methodology how to create an oscillator suitable for the modeling of the nonlinear dynamics. Upcoming chapters provide a general overview on the ideas behind the integrator based synthesis as well as classical circuit synthesis. Several examples of the autonomous deterministic dynamical systems with possible chaotic behavior implemented as an electronic circuits (chaotic oscillators) are also given. Each configuration is experimentally verified by a gallery of the oscilloscope screenshots as well as by a comparison of these pictures with numerical integration using Mathcad and build-in fourth-order Runge-Kutta method. It is worth nothing that Pspice circuit simulator give us the same results.

THE DISSERTATION THESIS IS AT ONE´S DISPOSAL

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1 INTRODUCTION

Chaos can be roughly considered as a long-term unpredictable behavior and resembles noise in many aspects. The typical feature of the chaotic circuit is its sensitivity to the tiny changes of the initial conditions and broad-band continuous frequency spectrum. This can be useful in many practical applications, especially for securing communication channels, for spreading data sequence or for a generation of the random numbers. Another properties [1] are ergodicity, fractal dimension, dense attractor, etc. The recent progress in the field of chaos theory is closely related with advances in the personal computer technologies and associated increase of the computer performance. By huge numerical simulation it has been discovered [2] that chaos in not restricted to the high-dimensional and the strongly nonlinear dynamical systems but can be observed even in the cases of simple systems with six terms including a quadratic nonlinearity. It is necessary to realize that if we are speaking about chaos we are also speaking about a certain set of internal system´s parameters and initial conditions (inside a basin of attraction) for its evolution simultaneously.

Electronic circuits are the mighty tools for studying nonlinear dynamics. A simple circuitry known as a Chua’s oscillator [3] was also the first time when chaotic motion has been experimentally confirmed. Nowadays it is well known that chaos can be observed also inside of the power electronics devices, digital circuits, phase locked loops, DC/DC converters, motor drivers, microwave circuits or optics. Thanks to the limited length of this work it is devoted to the various low frequency oscillators working in voltage, current or hybrid mode. By utilizing the current mode integrated circuits (IC) allows an engineer to create an oscillator ready for the higher frequency applications as demanded for telecommunication purposes.

If we ignore indisputable importance of chaos it is unwanted phenomenon in many applications. For example, a parasitic properties of the used electronic devices can cause such a situation. Even the notion that every harmonic oscillator is potentially chaotic can be found in some publications [4]. Taking into account some mechanism for the stabilizing oscillations which must be present we can adopt this proposition. Of course, harmonic oscillator itself is the second-order autonomous dynamical system but it is some sort of idealization without considering parasitic properties of the used active elements. From this point of view, a gallery of very interesting structures of the Wien-type feedback oscillators generating chaotic signal have been published [5]. It is shown that adding a single storage element is enough to generate a chaotic waveform. The authors in [6] present the chaotic oscillator consisting of interconnection of the nonlinear device (single diode) with a parallel LC resonant tank and a frequency dependent nonlinear resistor.

Chaos is up to date also from the theoretical standpoint. It is an universal phenomenon which was reported from many distinct scientific fields. At the beginning we are handling with the system of dimensionless differential equations thus a physical interpretation of the state variables or system parameters are unimportant.
2 CIRCUITRY IMPLEMENTATIONS

There are two different approaches to the synthesis of the nonlinear dynamical systems. Both can be used as soon as the mathematical model together with the ranges of the system’s parameters are given. It follows from the basic mathematics that a single higher-order differential equation can be easily recasted into desired form by introducing new state variables as the first derivatives of the other ones.

First possibility is an integrator based synthesis leading to the quite complicated final circuits with a lot of active components. Such a network can be easily extended to higher-dimensions for the chance of the hyperchaos. The main advantage of this conception is also straightforward design procedure with only three building blocks. It is an integrator (mainly inverting), summing or differential amplifier and a two-port with nonlinear characteristics with desired transfer shape. A design engineer can use a fundamental networks for these mathematical operations. An example of the voltage mode circuit is given in chapter 3 (universal oscillator) and chapter 4 (modeling weather circulation). The novel structure of the hyperchaotic system based on the Nosé-Hoover’s model of the thermostat is shown in the publication [7]. A truly current mode circuit well suited for IC implementation is presented in chapter 5.3. The integrator based synthesis has been used to design two wideband signal generators [8] with the slightly different vector fields. Synthesis based on the integrator block schematic can be used for the construction of hyperchaotic [9], conservative or optimized [10] oscillators. Following the same approach is an idea behind a current mode circuitry indicated by a diagram in [11]. The integrator based approach is often used for the generation of the multi-directional multi-scroll chaotic attractors [12] where this synthesis realize the entire linear part of the vector field.

Second design approach directly follows the rules of classical circuit synthesis. The core part is already well known and widely published. The application of this theory is summarized in the detailed article [13] together with some examples like Lorenz, Rossler, hyperchaotic system, Chua’s or forced van der Pol oscillator. Two ways leading to the same final circuits were proposed. One is based on the direct realization of the differential equations using operational amplifiers and successive substituing the typical network structures with elemental equivalent devices like ideal or lossy and grounded or floating accumulating two-port elements. Second possibility is in that the linear part of the vector field and the nonlinear function can be implemented simultaneously as an independent two-ports. Then, connecting both network in parallel and applying first Kirchhoff’s rule, one can easily get final oscillator working in the hybrid voltage-current mode. This is indeed especially profitable if the so-called nonlinear oscillator described by a single third-order differential equation with scalar nonlinear feedback function has to be realized. In the publication [14] Antoniou’s general impedance converter (GIC) has been used as a core engine. The simplest examples of such circuits with piecewise-linear (PWL) and polynomial vector field is given in [15]. Finally the one-dimensional multi-scroll chaotic attractors providing up to ten spirals is covered by chapter 6.
Chaotic oscillators proposed in this work have been constructed by using several active building blocks. Two types of the voltage-feedback operational amplifiers (AD713, TL084) with differential input and non-symmetrical output were employed in the case of voltage mode circuits with high impedance nodes. Modern active devices with some low impedance nodes are represented by a current feedback amplifiers (AD8002, OP260) and positive (AD844) or adjustable negative (EL2082) second generation current conveyors (CCII±). The operational transadmittance amplifiers (LM13700) with high gain-bandwidth product are another useful block for the higher-frequency oscillators. With the great progress in IC layout and design procedures other interesting multi-port building blocks are commercially fabricated, for example voltage mode four-quadrant single-channel (AD633) or four-channel (MLT04) multiplier and its current-mode single-channel equivalent (EL4083).

3 UNIVERSAL CHAOTIC OSCILLATOR

The integrator based synthesis has been used for the design of universal chaotic oscillator. The photograph of practical implementation using the SMD components is shown in Fig. 3.1. The final circuit is shown in Fig. 3.2 and some oscilloscope HP54603B screenshots are illustrated by means of Fig. 3.3. Having this circuit it is possible to model a full scale of dynamical events of both class C and F family [16], covered by the following system of equations

\[
\begin{align*}
-du_1/d\tau &= \varepsilon_{11}u_2 + \varepsilon_{12}[h(u_1 + \varepsilon u_2 + u_3) - u_3] + \varepsilon_{13}[u_1 + u_3 - h(u_1 + \varepsilon u_2 + u_3)] \\
-du_2/d\tau &= \varepsilon_{21}u_1 + \varepsilon_{22}u_2 + \varepsilon_{23}[h(u_1 + \varepsilon u_2 + u_3) - u_3] \\
-du_3/d\tau &= \varepsilon_{31}[h(u_1 + \varepsilon u_2 + u_3) - z] + \varepsilon_{32}h(u_1 + \varepsilon u_2 + u_3)
\end{align*}
\]

(3.1)

where \(\tau\) is given by the time constant same for each integrator.

![Fig. 3.1: The photograph of the laboratory device, universal chaotic oscillator.](image)

The individual parameters \(\varepsilon_{ij}\) can be adjusted independently, continuously and through the wide range of values (positive or negative). On the other hand, manual adjustment must be precise and this need a patience and consumes a lot of time. Mentioned adjustment is possible due to the external source of sine wave which is connected to the circuit via couple of switches S. The breakpoints Bp of the three-segment PWL transfer function are given directly by an external sources of dc voltage. This allow study the effect of non-symmetry on the particular attractors.
Fig. 3.2: Circuitry diagram of the universal chaotic oscillator.

The listing of passive circuit components for practical implementation is $C=33\text{nF}$, $R=47\text{k}\Omega$, $R_t=1\text{k}\Omega$, $R_s=10\text{k}\Omega$, $R_x=15\text{k}\Omega$. The practical realization of the active double-sided diode limiter is done by using two standard diodes 1N4148. It has been verified that using a passive diode-limiters leads into an expressive changes of some chaotic attractors. Finally, twenty-eight voltage-mode operational amplifiers TL084 are employed resulting into the necessity of seven packages. Presented oscillator can also generate harmonic, multiple harmonic or quasiperiodic signal. Using off-the-shelf components keeps the final price very low.
The double-scroll attractor can be observed for $\varepsilon_{11}=-1.38$, $\varepsilon_{12}=0.37$, $\varepsilon_{13}=-0.36$, $\varepsilon_{21}=1.38$, $\varepsilon_{22}=-0.36$, $\varepsilon_{23}=-0.25$, $\varepsilon_{31}=-2.63$, $\varepsilon_{32}=-1$, $\varepsilon_{33}=-2.91$, dual double-scroll attractor arise when $\varepsilon_{11}=2.59$, $\varepsilon_{12}=-0.21$, $\varepsilon_{13}=0.99$, $\varepsilon_{21}=-2.59$, $\varepsilon_{22}=0.99$, $\varepsilon_{23}=-2.02$, $\varepsilon_{31}=2.26$, $\varepsilon_{32}=3.98$, $\varepsilon=0.59$ and double-hook attractor can be observed if $\varepsilon_{11}=1$, $\varepsilon_{12}=8.79$, $\varepsilon_{13}=-0.18$, $\varepsilon_{21}=-1$, $\varepsilon_{22}=-0.18$, $\varepsilon_{23}=-19.53$, $\varepsilon_{31}=-1.04$, $\varepsilon_{32}=-1.25$, $\varepsilon=0$. These numbers hold for the state matrices in inner and both outer segments of the vector field in the so-called complex decomposed form [17] but any other form (elementally canonical, low triangular, etc.) can be modeled by this general oscillator. There are much more $\varepsilon_{ij}$ configurations leading to the various shapes of the chaotic attractors. Some of them can be taken for example from the publication [18].

Fig. 3.3: The plane projections of chaotic signals, universal chaotic oscillator.
4 MODIFIED LORENZ SYSTEM

The classical Lorenz system [19] is considered as the very first dynamical system where chaos has been confirmed. As it is mentioned in [20] the so-called modified Lorenz system can be used as a mathematical model of the atmosphere weather circulation. Detailed meaning of the state space variables and the internal system’s parameters are given in the article. This is a three-dimensional autonomous deterministic dynamical system described by a following set of the equations

\[
\begin{align*}
- \frac{dx}{dt} &= -\sigma x - y^2 - z^2 + \mathcal{F}, \\
- \frac{dy}{dt} &= x(y - N z) + \phi, \\
- \frac{dz}{dt} &= -z + x(N y + z),
\end{align*}
\]

(4.1)

where $\sigma$, $N$, $\mathcal{F}$ and $\phi$ are real-valued parameters.

Fig. 4.1: Contour plot of the largest LE of the modified Lorenz system, see text.
It is not hard to find out that searching for equilibria leads through solving a fifth-order polynomial so that up to five real solutions are possible. Following the rules of linear algebra we can easily find out that depending on the parameter’s values the corresponding eigenvalues can be both one real together with a pair of complex conjugated or all three real. However this analysis does not give us any idea about parameter’s regions for chaotic behavior. From the viewpoint of such solutions the planes $\vartheta \in (0.3, 2)$ and $\Im \in (1, 9)$ has been deeply studied using the concept of the spectrum of the Lyapunov exponents (LE). The largest LE indicating the possible occurrence of chaos is shown in Fig. 4.1 using the grayscaled contour plot with scale $\text{LE}_{\text{max}} \in (-0.01, 0.75)$. During the practical measurement of the chaotic oscillator below the unpublished bounded region of chaos has been discovered.

Fig. 4.2: Contour plot of the largest LE of the modified Lorenz system, see text.
This area is given by the following ranges of parameters $\vartheta \in (1, 1.7)$ and $\mathfrak{I} \in (1, 1.5)$ and its shape is visible by means of Fig. 4.2 with scale $\text{LE}_{\text{max}} \in (-0.01, 0.24)$. It is evident that the most common solution of the modified Lorenz system within given ranges of $\vartheta$ and $\mathfrak{I}$ is a limit cycle. There are just three LE and each is a real number giving the average ratio of exponential divergency of the two neighborhood trajectories. Since one LE must be close to zero (direction of the flow) to obtain sensitivity to the initial conditions (chaos) it is necessary to have one LE positive. The last LE must be negative with the largest absolute value to preserve dissipation.

Fig. 4.3: Circuitry diagram of the modified Lorenz system.
The concrete values of the passive elements of the developed circuitry shown in Fig. 4.3 are $C=100\text{nF}$, $R_5=R_{11}=R_{12}=R_{17}=1\text{k}\Omega$, $R_2=R_{14}=20\text{k}\Omega$, $R_3=R_7=R_9=40\text{k}\Omega$, $R_8=R_{18}=R_{21}=100\text{k}\Omega$ and the rest of the resistors are fixed at $100\text{k}\Omega$. The photograph of laboratory device is depicted in Fig. 4.5 and selected oscilloscope HP54603B screenshots are shown in Fig. 4.4. For necessary mathematical operations eight standard operational amplifiers AD713 were used (only two packages are needed). Potentiometers $1\text{M}\Omega$ serves for independent adjusting of the parameters $\vartheta$, $\wp$ and product of parameters $\vartheta \wp$. Note that two symmetrical supply voltages are essential, namely $\pm 15\text{V}$ and $\pm 5\text{V}$. The transfer function of the multiplier MLT04 is $W=KXY$ where $K=0.25$ and allows to multiply signals up to $2.4\text{V}$. To imagine how the chaotic signal in time domain look-like there is an example in Fig. 4.6.

![Fig. 4.4: The plane projections of chaotic attractors, modified Lorenz system.](image)
For a comparison also the waveform of the typical attractor (double-scroll) of the universal oscillator is given via Fig. 4.7. More details about practical measurement are given in [21]. Laboratory experiments also confirm its the broad-band frequency spectrum. For the theoretical study it is much interesting to get the bifurcation sequences by using repeated numerical integrations. The corresponding results are provided in [22].

Fig. 4.5: The photograph of the laboratory device, modified Lorenz system.

Fig. 4.6: Chaotic signals, two different attractors of the modified Lorenz system.

Fig. 4.7: Chaotic signals, universal oscillator.
5 NONLINEAR OSCILLATOR

The classical circuit synthesis is ideal for construction of the nonlinear oscillator based on the single third-order differential equation of the form
\[ \ddot{x} + \varphi_1 \dot{x} + \varphi_2 x = g(x), \] (5.1)
where dots denote the derivatives with respect to the independent variable, time. Since the individual state variables can be interpreted as a position, velocity and acceleration such a dynamical system can model particular situations in mechanics. Due to the dissipation the restriction \( \varphi_1 > 0 \) must be satisfied. It is shown in [23] that the nonlinear feedback \( g(x) \) can be polynomial, exponential or even goniometrical function. In spite of this, the most attractive for practical implementation is a two segment PWL shape. Since also \( \varphi_2 > 0 \) it is evident that at least one segment must be negative to get unstable equilibria. Note that for such nonlinear AV characteristics some current-providing two-port connected with single diode is needed. Two fixed points in these circuits are possible only if current source is presented. The signum function can be easily represented by a single comparator. A very simple network can be created also for the resistor with polynomial AV curve. The single-scroll or funnel-like chaotic attractor can be generated by a quadratic nonlinearity, for the double-scroll attractor a cascade connection of two multipliers (cubic nonlinearity) is necessary. There exist several cases of the function \( g(x) \) providing conservative chaotic dynamics \( \varphi_1 = 0 \). Some of them with PWL vector field are verified in [24].

5.1 VOA BASED STRUCTURES

The linear part of the vector field can be implemented by selecting one circuit from the Fig. 5.1 with proper values of the circuit elements. It is straightforward process to obtain the input admittance function in the general form
\[ Y_{in}(s) = \frac{I_{in}(s)}{U_{in}(s)} = \frac{a_3 s^3 + a_2 s^2 + a_1 s + a_0}{b_2 s^2 + b_1 s + b_0}, \] (5.2)
of the both structures. In this equation \( I_{in}(s) \) and \( U_{in}(s) \) are a Laplace transforms of the input current and voltage respectively. For the first network known as the Antoniou’s general impedance converter (GIC) one can get easily
\[
\begin{align*}
    a_3 &= c_1 c_2 c_3 r_1 r_2 r_3 (\Im_1 \Im_2 + \Im_1 + 1) \\
    a_2 &= c_1 c_2 r_1 r_3 (\Im_2 + 1) + c_1 c_2 r_2 r_3 (\Im_1 + 1) + c_1 c_2 r_1 r_2 (\Im_1 \Im_2 + \Im_1 + 1) \\
    a_1 &= c_1 (\Im_2 + 1)(r_2 + r_3) \\
    a_0 &= 0 \\
    b_2 &= c_2 c_3 r_1 r_2 r_3 (\Im_1 + 1) \\
    b_1 &= c_3 r_2 r_3 (\Im_2 + 1) + c_2 r_1 (\Im_1 + 1)(r_2 + r_3) \\
    b_0 &= (\Im_2 + 1)(r_2 + r_3) + \Im_1 \Im_2 r_3
\end{align*}
\] (5.3)
Assume that the gain \( \Im_1(\omega) \) as well as \( \Im_2(\omega) \) approaches infinity which is a good approximation for the voltage-feedback operational amplifier (VFA). Also consider a linear capacitor \( c_4 \) connected in parallel with GIC. This leads to the simplification
\[
\begin{align*}
    a_3 &= c_1 c_2 c_3 r_2 \\
    a_2 &= c_1 c_2 r_2 r_3 / r_3 \\
    a_1 &= c_4 \\
    a_0 &= 0 \\
    b_2 &= b_1 = 0 \\
    b_0 &= 1.
\end{align*}
\] (5.4)
Using the same computation method for the second one we get
\[
    \begin{align*}
        a_3 &= c_1c_2c_3r_1r_2r_3(3 + 1) \\
        a_2 &= c_1c_2r_1(r_1 + r_2)(3 + 1) + c_3r_2[c_1r_3 + c_2(r_1 + r_3)] \\
        a_1 &= r_3[c_1 + c_2(3 + 1) + c_3] + c_2(r_1 + r_2) \\n        a_0 &= 0 \\
        b_2 &= c_2c_3r_1r_2r_3 \\
        b_1 &= r_3[c_2 + c_3] + c_2r_1 \\
        b_0 &= r_3(3 + 1)
    \end{align*}
\]
(5.5)

It the gain \( \Im(\omega) \) is high enough these coefficients simplifies into
\[
    \begin{align*}
        a_3 &= c_1c_2c_3r_1r_2r_3 \\
        a_2 &= c_1c_2(r_1 + r_2) \\
        a_1 &= c_1 + c_2 \\
        b_2 &= b_1 = 0 \\
        b_0 &= 1
    \end{align*}
\]
(5.6)

The main advantage of both inductorless admittance networks are the similar relations between the system parameters and circuit elements. By comparing (5.4) with (5.1) these terms are
\[
    c_1 = c_2 = c_3 = 1 \quad c_4 = \varphi_2 \quad r_1 = r_2 = 1 \quad r_3 = 1 / \varphi_1.
\]
(5.7)

For the second case we should compare (6.6) with (6.1) resulting into
\[
    \begin{align*}
        c_1 &= c_2 = \varphi_2 / 2 \\
        c_3 &= \left(\varphi_2 / \varphi_1\right)^2 \\
        r_1 &= r_2 = 2\varphi_1 / \varphi_2^2.
    \end{align*}
\]
(5.8)

As mentioned before the simplest nonlinear resistor has the signum-type \( AV \) curve. This can be intuitively implemented as it is shown by the first network in Fig. 5.2. Starting form of the nonlinear function and corresponding circuit elements are
\[
    g(x) = \xi \text{ sign}(x) - \varphi_3x \quad \Rightarrow \quad r_a = V_{sat} / (\varphi_3V_{sat} - \xi) \quad r_b = V_{sat} / \xi.
\]
(5.9)

The complete \( AV \) curve for this resistor is the first picture shown in Fig. 5.3. As it is discussed in [25] we can theoretically construct polynomial resistor up to an almost arbitrary degree by a proper number of the multipliers binded together. Second circuit shown in Fig. 5.2 can be directly used as the simplest quadratic resistor.

Fig. 5.1: Two implementations of the third-order admittance function.
Due to the transfer function of a multiplier AD633 designed to realize polynomials decomposed into the root products in the form \( W = K(X_1 - X_2)(Y_1 - Y_2) + Z \) where \( K = 0.1 \) it is not hard to derive term for resistor

\[
g(x) = \pm \left( \kappa x^2 - \eta \right) \Rightarrow r_a = K / \kappa.
\]  

(5.10)

It is obvious that a constant current source (CCS) should be connected in parallel. CCS can be realized by adding dc voltage source in series with fixed resistor or by a slight modification of multiplier’s input nodes interconnection. In detail, if one external dc voltage source \( V_a \) is connected to node \( Y_1 \) and another dc voltage \( V_b \) to node \( Z \) then \( g(x) \) is directly implemented with no other circuit components needed. Of course, \( V_a \) and \( V_b \) must be adjusted carefully to discard the linear term in the nonlinear AV characteristics. If \( V_a = 10V \) then \( r_a \) can be dropped and \( V_b = \eta / \zeta \). First measured AV curve shown in Fig. 5.3 belongs to the signum-type resistor, second screenshot is the fundamental configuration of the polynomial resistor and the last picture is a modified polynomial resistor. A current-sensing resistor and auxiliary triangular signal has been used for this purpose.

For a given set system parameters \( \varphi_1 = 0.6, \varphi_2 = 1, \varphi_3 = 1.2, \xi = 1 \) and \( \zeta = 2 \) the circuit components for GIC are \( c_1 = c_2 = c_3 = c_4 = 10nF, \quad r_1 = r_2 = 10k\Omega, \quad r_3 = 17k\Omega, \quad r_a = 9.6k\Omega \) and \( r_b = 65k\Omega \). Analogically for the canonical admittance we get \( c_1 = c_2 = 5nF, \quad c_3 = 27nF, \quad r_1 = r_2 = 12k\Omega, \quad r_a \) and \( r_b \) remain the same. The plane projections of individual chaotic attractors typical for both admittances and resistors are shown in Fig. 5.4, Fig. 5.5, Fig. 5.6 and Fig. 5.7. The experimental observations prove that using VFA as a comparator is possible only for lower frequency applications due to the hysteresis.

Fig. 5.2: The signum-type and polynomial-type nonlinear resistor.

Fig. 5.3: Measured AV curve for signum-type and polynomial-type resistors.
Note that the state variables of the original mathematical model (5.1) should be first and second derivatives of the voltage. These are not directly represented by a node voltage neither by some current. Both admittances have two independent node voltages which are a linear combination of the state variables. Thus the state space attractors are measurable. In spite of this, sometimes it is useful to obtain original state space representation. This task can be done by a cascade connection of the admittance $Y_1(s)=s^3$, $Y_2(s)=Ds^2$ and $Y_3(s)=Cs$. We can get first admittance using GIG with omitted $R_3$ and second admittance using the same circuit with omitted $C_3$. In this case the individual state variables are current flowing to both GICs which are transformed into node voltage. The resulting strange attractors are shown in Fig. 5.8.

The integrator based synthesis of the nonlinear oscillator with various nonlinear feedback functions can be found in [26]. It seems that there is no need to use three VFAs for the integration since one integrator can be passive.

![Fig. 5.4: Chaotic attractors, first admittance and signum-type resistor.](image)

![Fig. 5.5: Chaotic attractors, first admittance and polynomial-type resistor.](image)

![Fig. 5.6: Chaotic attractors, second admittance and signum-type resistor.](image)
Fig. 5.7: Chaotic attractors, second admittance and polynomial-type resistor.

Fig. 5.8: The plane projections of chaotic attractors, cascaded oscillator.
5.2 CCII BASED STRUCTURES

Before speaking about particular circuitry implementations the three-port known as a general current conveyor (GCC) should be introduced. It is a modern functional block ideally described by a hybrid matrix

\[ U_x = \alpha U_y \quad I_y = \beta I_x \quad I_z = \gamma I_x. \] (5.11)

By using a floating nullor concept we can replace VFA by a CCII. Before doing this let consider admittance network with GCC shown in Fig. 5.9 on the left. This two-port has the quite complicated input admittance

\[ a_3 = c_1 c_2 c_3 r_1 r_2 \quad a_2 = c_2 [c_1 (r_1 + r_2) + c_3 r_1 (1 + \gamma)] \]
\[ a_1 = c_1 + c_2 + c_3 [1 - \alpha (1 + \beta + \gamma) + \beta + \gamma] \quad b_2 = -c_1 c_3 r_1 r_2 \alpha \beta. \] (5.12)
\[ b_1 = -\alpha \beta [c_1 (r_1 + r_2) + c_3 r_1] \quad b_0 = -\alpha (\beta + \gamma) \]

To get the positive (energy consuming) admittance CCCII- \((\alpha=1, \beta=0)\) should be employed leading to the following simplification

\[ a_3 = c_1 c_2 c_3 r_1 r_2 \quad a_2 = c_2 [c_1 (r_1 + r_2) + c_3 r_1 (1 + \gamma)] \quad a_1 = c_1 + c_2 \quad b_0 = -\gamma. \] (5.13)

Since \(\gamma \in (-2, 0)\) is a function of the gain voltage \(V_g\) we can get admittance (5.6) by setting \(\gamma = -1\). It is evident that if the fully electronically adjustable chaotic oscillator with three system parameters is desired we are in need of at least three CCCII-.

Assume the parallel connection of three admittances

\[ Y(s) = \tilde{Y}(s) + \tilde{Y}(s) = \alpha \tilde{s}^3 + \hat{a}_2 \tilde{s}^2 + \hat{a}_1 \tilde{s} \quad \tilde{Y}(s) = \tilde{\alpha}_1 \tilde{s} \]
\[ Y(s) = \tilde{Y}(s) + \tilde{Y}(s) = \alpha \tilde{s}^3 + (\tilde{a}_2 + \hat{a}_1) \tilde{s}^2 + (\tilde{a}_1 + \hat{a}_1 + \hat{a}_1) \tilde{s}. \] (5.14)

First admittance is given by (5.13) and the second one can be considered as a lossy frequency dependent negative resistor (FDNR). For the particular schematic shown in Fig. 5.9 and GCC we get the input admittance

\[ \tilde{Y}(s) = -[c_4 c_3 r_3 \tilde{s}^2 + (c_4 + c_3) \tilde{s}] / [c_3 r_3 \alpha \beta s + \alpha (\beta + \tilde{\gamma})]. \] (5.15)

The input admittance for CCII- becomes

\[ \tilde{Y}(s) = -[c_4 c_3 r_3 \tilde{s}^2 + (c_4 + c_5) \tilde{s}] / \tilde{\gamma}. \] (5.16)

Fig. 5.9: The third-order admittance, adjustable FDNR and variable capacitor.
Finally the adjustable capacitor is also given in Fig. 5.9. If GCC is utilized for such purpose its input admittance is in the form
\[ \tilde{Y}(s) = -c_0 s / [\alpha (\beta + \gamma)] \rightarrow \tilde{Y}(s) = -c_0 s / \gamma. \] (5.17)

By substituting (5.13), (5.16) and (5.17) into (5.14) we get immediately
\[ Y(s) = c_1 c_2 r_1 r_2 s^3 + [c_1 c_2 (r_1 + r_2) - c_4 c_5 r_3 / \gamma] s^2 + [c_1 + c_2 - (c_4 + c_5) / \gamma - c_6 / \gamma] s. \] (5.18)

Taking into account the final equation (5.18) and comparing individual coefficients with (5.1) a little computation gives the following listing of the circuit elements
\[ c_1 = c_2 = \varphi_2 / 8 \quad c_3 = \varphi_2^2 / (4\varphi_1^2) \quad c_3 = c_4 = \varphi_2 / 8 \quad c_5 = \varphi_2 / 2 \]
\[ r_1 = r_2 = 16\varphi_1 / \varphi_2^2 \quad r_3 = 32\varphi_1 / \varphi_2^2. \] (5.19)

Fig. 5.10: Negative resistor and PWL resistor with two segments.

Fig. 5.11: One-dimensional bifurcation diagrams, see text.

Fig. 5.12: One-dimensional bifurcation diagrams, see text.
Two-segment PWL resistor is shown in Fig. 5.10. This network allows us to have different slopes in both segments as well as to adjust the offset of the nonlinear function. The main question is if we are able to study various routes to chaos via some control parameter $\gamma$ of the individual CCCII-. This can be answered by visualisation of the one-dimensional bifurcation diagrams constructed by means of succession of the Poincaré sections defined as $\{\Pi: y=0, x \in \mathbb{R}\}$. The transition events are omitted (computation starts when trajectory reach the attractor) and high precision is acquired using small integration step $\Delta t=0.001$. The parameter space for Fig. 5.11 is following

$$\tilde{\gamma} \in (-2, -0.2)$$ $\hat{\gamma} = 1$ and $\hat{\gamma} = -1$ $\tilde{\gamma} \in (-1.1, -0.5)$, (5.19)

similarly for the Fig. 5.12 the original parameter space is

$$\tilde{\gamma} \in (-2, -0.2)$$ $\hat{\gamma} = -0.8$ and $\hat{\gamma} = -0.8$ $\tilde{\gamma} \in (-1.1, -0.5)$, (5.20)

analogically Fig. 5.13 results from the choice

$$\tilde{\gamma} \in (-0.6, -0.2)$$ $\hat{\gamma} = -1.2$ and $\hat{\gamma} = -1.2$ $\tilde{\gamma} \in (-1.1, -0.5)$. (5.21)

Clearly almost any adjustment of the individual parameters of equation (5.1) leads into some bifurcation sequence which is observable by a laboratory measurement. This proposition can be useful for the digital control of these parameters. On the other hand the certain resolution and step-values of the digital potentiometers can make such an oscillator difficult to realize. Note that the parameters of the nonlinear function can be used for influencing the size of the attractor as shown in Fig. 5.14.
5.3 GM-C CHAOS

It is obvious that from the viewpoint of practical applications it is convenient to design an oscillator with structure easy to be implemented on chip. Since the main aim of the IC designer is to save the space on chip for the particular functional block the chaotic oscillator we are looking for has to be resistorless. This is possible if the entire oscillator is working in the current-mode as it is suggested in Fig. 5.16. It is well known that the summing operation in the current-mode integrator-based circuits is done simply by a node which is of great advantage. Note that there are three operational transadmittance amplifiers (OTA) with multiple outputs. Unfortunately this building block is not commercially available as IC. Although it is possible to realize several current outputs by a cascading the current mirrors this makes the final oscillator’s structure much more complicated. In spite of this the IC implementation of this to date hypothetical component is quite easy. The constant current source can be derived from the reference voltage which is allways presented on chip.

If we are thinking about practical implementation of the current-mode chaotic oscillator each resistor can be also substituted by a discrete OTA with programmable transadmittance. It is necessary to achieve a linear relation between transadmittance and the control current for these OTAs in the real circuit. If this is not satisfied the one can be unable to discover particular state space attractors. The reason for this is follows from Fig. 5.15. This contour plot of the largest LE has the high resolution $\Delta \delta_1=\Delta \delta_2=0.001$ uncovering how the structure of the chaotic attractor is sensitive to the changes of two adjustable parameter. In this picture the two-dimensional parameter space under inspection is $\delta_1 \in (-0.55, -0.45)$ and $\delta_2 \in (-2, -1.85)$. In the case of $\delta_1$ greater than -0.45 the solution becomes unbounded. It should be noted that only limit cycles and chaotic regions are possible for this interval of parameters.

Fig. 5.15: Contour plot of the largest LE $\in (-0.02, 0.15)$ as a function of $\delta_1$ and $\delta_2$. 
The rest of OTAs together with the linear capacitors form the current integrators. The entire structure is somehow familiar to the so-called follow the leader network with input summation known from the theory of electronic filters. The time constant of the circuit is supposed to be varied simultaneously. The simple final circuitry implies that the original mathematical model was algebraically trivial, namely it was the case denoted as JD2 from the publication [2].
6  MULTI-SCROLL CHAOS GENERATOR

Assume the mathematical model (6.1) coupled together with the nonlinear stair-type scalar function

\[ g(x) = a_0x + \gamma + \sum_{i=1}^{\psi} a_i |x - \beta_i|, \tag{6.1} \]

where \( \psi \) is a number of breakpoints of the PWL function. It seems that the most suitable practical implementation is to use the classical circuit synthesis, i.e. parallel combination of the higher-order admittance function and PWL resistor. This resistor consists of the parallel connection of \( \psi \) comparators and a current summation block as shown in Fig. 6.1. The individual breakpoints can be adjusted independently on each other by external dc sources. The voltage levels at the outputs of the VFAs working as the comparators are given by the supply voltages (for \( \pm 15V \) supply voltage this level is \( \pm 13V \)). These voltages cause currents which are summed by a CCII+. Note that there is no need to transfer output current into voltage as it is required in the case of integrator based synthesis. For the general shape of the stair-type nonlinearity we should establish the values of \( R_i \) by solving the system of linear nonhomogenous equations. Fortunately the nonlinearity is symmetrical so that each \( R_i = U_{sat}/\Delta I \) where \( \Delta I = 100\mu A \) represents a basic current step. The final AV curves for four, six and eight-scroll attractors and measured by means of the current-sensing resistor and auxiliary triangular signal are demonstrated in Fig. 6.2. The generation of multi-spirals attractors are demonstrated via Fig. 6.3 (four, six and eight spirals).

Fig. 6.1: The stair-shaped nonlinear PWL resistor with four breakpoints.
The last plane projection is almost the same for arbitrary number of the spirals since each spiral lie on the x axis. This event is visible in Fig. 6.4. Fig. 6.5 shows how the multi-spiral chaos in the time domain looks like. The limited dynamical ranges of the used active elements (output voltage and/or current saturation) restrict also the possible number of the generated spirals. In our case, up to ten spirals have been experimentally confirmed. Using PWL functions with multiple variables can lead to multi-scrolls attractors linked in two or three dimensions.

Fig. 6.2: Measured AV characteristics of the nonlinear resistors.

Fig. 6.3: Generation of the multi-spiral chaos, xy and xz plane projections.

Fig. 6.4: Generation of the multi-spiral chaos, yz plane projection.
Fig. 6.5: The chaotic signal corresponding to four-scroll attractor in time domain.

Also goniometrical function can be directly used for the purpose of the multi-spiral chaos generation, see [27] for further details.

7 RC OSCILLATOR WITH GENERAL FEEDBACK

There are many dynamical systems which can be written in the following compact matrix form

\[
\dot{x} = A x + b \, h(w^T x),
\]  

(7.1)

where \(A\) is \(3 \times 3\) square matrix, \(b\) and \(w\) are \(3 \times 1\) column vectors and \(h(.)\) is a scalar PWL function. The expression (7.1) covers class C and F as well as the harmonic oscillator with RC ladder feedback introduced in [28]. Thus it is useful to discover an universal circuit capable to model any dynamics associated with (7.1). One such oscillator is demonstrated in Fig. 7.1. This circuit consists of the linear capacitors and resistors, three summing (differencing or inverting) amplifiers with two inputs and output voltages

\[
u_{s1} = s_{12} u_2 + s_{13} u_3 \quad u_{s2} = s_{21} u_1 + s_{23} u_3 \quad u_{s3} = s_{31} u_1 + s_{33} u_2,
\]  

(7.2)

and four-port with PWL transfer characteristics. It follows directly from the circuit that if \(s_{ij}=0\) and \(s_{ik}=0\) then \(u_{si}=0\) and the corresponding summing amplifier can be removed. The individual state variables are always voltages across linear capacitors. The entire dynamical system is described by the dimensionless differential equations

\[
\frac{d}{dt} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} -d_1 (g_{1} + g_{s1} + g_{b1}) & d_1 g_{s1} s_{12} & d_1 g_{s1} s_{13} \\ d_2 g_{s2} s_{21} & -d_2 (g_2 + g_{s2} + g_{b2}) & d_2 g_{s2} s_{23} \\ d_3 g_{s3} s_{31} & d_3 g_{s3} s_{32} & -d_3 (g_3 + g_{s3} + g_{b3}) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \begin{pmatrix} d_1 g_{b1} \\ d_2 g_{b2} \\ d_3 g_{b3} \end{pmatrix} \begin{pmatrix} \Psi_0 \, w^T u_1 \\ \Psi_0 - \Psi_1 \\ 2 \end{pmatrix} + \begin{pmatrix} w^T u_1 \\ w^T u_2 \\ w^T u_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix},
\]  

(7.3)

where \(d_i=1/c_i\), \(g_{si}=1/r_{si}\), \(g_{bi}=1/r_{bi}\) and vector \(w^T=(w_1 \ w_2 \ w_3)\). The large amount of the circuit elements is essential because this allows us to choose normalized capacitors \(c_1=c_2=c_3=1\), \(\Psi_0=1\) and \(\Psi_1=0\) (the same transfer function as presented in Fig. 3.2) or to force floating resistors to be only positive. For some configurations of matrix \(A\) and vector \(b\) a few grounded negative resistors are necessary.
The negative resistor can be obtained by assuming first picture in Fig. 5.10. Another realization of the negative impedance converter consists of three resistors and single VFA [29]. This idea has been followed in our case. Note that the inputs of summing amplifiers are supposed to have infinite impedance which is impossible in any application except voltage followers. In practice, there are error terms in the main diagonal of the matrix $A$ which can seriously deform or destruct the desired chaotic attractor. Thus the resistors employed in the summing amplifiers must be high enough. The proposed circuit can be easily upgraded into four-dimensions for a chance of hyperchaos.

From the viewpoint of the implementation with minimum active blocks it seems that the circuit components should be computed by the following way. If $\text{sign}(b_i)=1$ and $\text{sign}(A_{ii}+b_i)=-1$ then

$$ g_i = 10^{-9} \quad g_{bi} = b_i \quad g_{si} = -(A_{ii} + b_i) \quad s_{ji} = -\frac{A_{ji}}{A_{ii} + b_i} \quad i \neq j . \quad (7.4) $$

In practice the admittance $10^{-9}$ can be considered as an open circuit and the nonzero value came out from the Mathcad program routine to preserve (7.1) being integrable. There is also the possibility to make a minor circuit transformation if $s_{ij}=s_{ji} \neq 0$ and $g_{si}=g_{sj}$. In this case one summing amplifier can be replaced by a single resistor.

![Fig. 7.1: The circuitry implementation of universal RC oscillator.](image-url)
This is a key factor to the proposition that universal RC oscillator also covers the configuration of the parallel connection of the third-order admittance RC network with three-segment PWL resistor. The values of the circuit elements are evident. If $\text{sign}(b_i)=1$ and $\text{sign}(A_{ii}+b_i)=1$ then

$$g_i = -2(A_{ii} + b_i) \quad g_{bi} = b_i \quad g_{si} = A_{ii} + b_i \quad s_{ij} = \frac{A_{ij}}{A_{ii} + b_i} \quad i \neq j.$$  \hspace{1cm} (7.5)

Fig. 7.2: The plane projections of the selected chaotic attractors.
Similarly to the previous cases if \( \text{sign}(b_i) = -1 \) and \( \text{sign}(A_{ii}+b_i) = -1 \) then
\[
g_i = 2(A_{ii} + b_i) \quad g_{bi} = -b_i \quad g_{si} = -(A_{ii} + b_i) \quad s_{ij} = \frac{A_{ij}}{A_{ii} + b_i} \quad i \neq j ,
\]  
and if \( \text{sign}(b_i) = -1 \) and \( \text{sign}(A_{ii}+b_i) = 1 \) then
\[
g_i = 10^{-9} \quad g_{bi} = -b_i \quad g_{si} = A_{ii} + b_i \quad s_{ij} = -\frac{A_{ij}}{A_{ii} + b_i} \quad i \neq j .
\]  
Finally if \( \text{sign}(b_i) = 0 \) then
\[
g_i = A_{ii} - 1 \quad g_{bi} = 10^{-9} \quad g_{si} = 1 \quad s_{ij} = A_{ij} \quad i \neq j .
\]
Some selected interesting state space attractors observed by using the proposed oscillator are given in Fig. 7.2. The concrete values of the circuit elements are not given but can be easily established by using formulas (7.4) to (7.8) and some set of the eigenvalues provided in [18].

8 CONCLUSION

This two-part of the monograph is aimed on the chaos-based topics. The first-part deals with the circuitry implementations of the selected dynamical systems. The attention is focused mainly on easy to understand design procedures and novel oscillator’s structures. A very simple one-dimensional multi-spiral chaos generator is presented and experimentally verified. As suggested in [30] such a circuit can be used for a generation of the random bit sequences. It seems that some engineers are interested in another tasks like to find chaotic behavior in some given network structure. The authors in [31] try to solve this topic for a simple RLC network with a single active block. The fundamental publication [32] solves the main problem of modeling piecewise-linear vector fields given by an eigenvalues by an electronic circuit. Another chaotic oscillator providing a strange structure of the state space attractor is described in [33]. Here the integrator-based synthesis has been used as it is also in the case of [34]. In this publication the class of algebraically simple system of three first-order differential equations with a single quadratic-type nonlinearity has been implemented as a general circuit. The general overview on the dynamics associated with the circuits with passive-only nonlinear devices are provided in the recent publication [35]. In these very simple oscillator’s topologies only single diode or unipolar transistor is used as a vent. The authors were able to separate individual configurations to subclasses with the same dynamical motion. Finally, many other nonlinear dynamical systems, interesting ideas and applications can be found in the book [36]. The systematic way how to create a circuit with associated chaotic behavior is outlined in [37].

Of course, there exist several fundamental publications about chaos utilization in the practical use. An idealized secure communication purpose is given in [38]. Much more realistic results are provided in [39] but at the cost of huge mathematical background. The publication [40] is important from the viewpoint of the circuit theory and can be also considered as a consequence of studying chaotic motion.
REFERENCES


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