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A Topological Approach

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1 INTRODUCTION

1.1 Domain of the Thesis

The principal subject of interest of this thesis are methods of image segmentation. What is generally understood by segmentation of an image is a transform permitting to extract the ‘objects’ contained in the image. The objects are identified by classifying the points of the image in various classes. These classes can be defined by using various criteria based on local or global light intensity, geometry, topology, statistic or other properties. More sophisticated methods use for the class identification combinations of these criteria or are based on some model of the object. According to the number of classes present in the image, we distinguish two-class and multi-class images. The two-class images usually contain one class for the object and one class for the background. Note, that objects of a two-class image may, however, comprise several components which are separated from each other by the background. The multi-class images are characterized by a different class for each object. Therefore, objects in a multi-class image are not necessarily separated by the background. If a light intensity criterion is used to define the image classes then the neighboring objects are distinguished by different values of the light intensity.

Many segmentation techniques have been proposed so far. Besides other criteria, they can be classified by their applicability to two-class or multi-class images. We are particularly interested in low-level, data-driven\textsuperscript{1} image segmentation techniques. Two principal, and entirely different approaches have been studied in the manuscript.

1.2 Structure of the Manuscript

1.2.1 Part I.

Part I references the basic concepts of image processing used throughout this manuscript. Basic topological notions are given with references and examples. Definitions of the neighborhood of a point, path, connected component, hole or cavity are presented in such a form they are usually found in the literature.

In the introductory section we also give a comparison of the most frequently used mathematical models for topology modeling. Two types of models are distinguished, as proposed by Lienhardt [Lie91]: the topological models and the embedding models. The topological models are used to describe the topology of space subsets, i.e. the mutual position, form, adjacency relation, etc., of various objects in the image. The embedding models are used for their embedding in a discrete space. See this part to refer to topological models such as the incidence graphs, combinatorial maps or

\textsuperscript{1}i.e., not controlled by any a priori model.
generalized combinatorial maps. The embedding models advanced in this part include the point-to-point model, cellular complexes and the Khalimsky grid.

1.2.2 Part II.

Part II concerns the image segmentation method called $h$-minima. Originally, the $h$-minima have been introduced within the framework of mathematical morphology (see Serra [Ser82] and [Ser88]). The $h$-minima is a low-level operator based on the local light intensity in the image. It is often used for finding the markers of the objects. These markers are in the following step of processing employed to initialize or guide the segmentation.

$h$-minima, being originally conceived to segment two-class images, its application to multi-class images was possible only indirectly by transforming the multi-class image into a two-class image. A multi-class image can be transformed to a two-class image by extracting the gradient of the light intensity function. Any uniform object is thus transformed to a dark region and any area of light intensity transition is represented by a high gradient value. In the gradient, there are only two classes: the dark areas belong to one class and every elevation of the gradient to the other class. The indirect, gradient-oriented approach is disadvantageous, since the extraction of the gradient filters out the detail from the image. In the manuscript of the thesis, we explain the drawbacks of the original gradient-based approach and propose a modification of the $h$-minima operator, which is directly applicable to multi-class images. The gradient extraction becomes useless, and all details in the image are preserved.

The performance of our modification of the $h$-minima operator is demonstrated on a 2-D grey-scale, multi-class image that, we believe, illustrates best the limitations of the original approach using the gradient extraction.

1.2.3 Part III.

Part III presents two topology-oriented, entirely three-dimensional segmentation methods. These methods are based on a homotopic deformation controlled by the light intensity. A continuous deformation in a discrete space is performed by using the simple points. The principle of these methods gives entire control over the topology of the result. These methods are advantageous in such applications where the topology of the result is important and fixed. The research has been motivated by the work of Luc Soler (see the thesis [Sol98]) at Inria, Sophia-Antipolis, France. The method introduced by Soler is based on light-intensity thresholding with subsequent topology and geometry correction of the result. The input image is a 3-D, grey-scale image where the object to extract is visualized by the contrast-enhanced X-ray tomography. The object to extract is the portal vein of the liver. Since the liver vascular system is an object of a considerable geometric complexity, the subsequent correction of the topology of the result is quite a complex task.
The objective of our interest in segmentation of this type of object is to eliminate
the necessity of subsequent topology correction. We propose two original, low-level
segmentation techniques and one method of extraction of a thin result (skeleton of the
object). Our methods control the topology during the processing. The result is guaran-
teed to be topologically correct and therefore the topology-correcting post-processing
becomes useless.

In the beginning of the Part III we give an introductory section concerned with
simple points. The introduction of the notion of a simple point has been historically
preceded by the crossing numbers examining the neighborhood. The character-
zation of simple points in 3-D has been done by using several techniques before
the complexity-optimal characterization based on binary decision diagrams has been
found (refer to [Bry86]). The historic and characterization aspects of simple points
are shortly presented in the beginning of the Part III.

After the introductory section, we present two segmentation methods and one skele-
tonization method. These methods are based on the principle of continuous object
deformation controlled by local light intensity, \( \lambda \)-thinning, \( \lambda \)-thickening and \( \lambda \)-ske-
letonization. These methods perform a successive deletion of simple points. The
two segmentation methods are dual. The skeletonization method is derived from \( \lambda \)-
thinning by a modification of the selection criteria selecting the simple points to be
deleted.

In the last sections of this part, we finally propose a parallel skeleton filtering
method which is used to noise cancellation in the skeleton. Parallel deletion of noisy
skeleton segments is not easy and reports to parallel deletion of simple points\(^2\). The
parallel deletion of noisy skeleton segments that we propose is based on the same
principle of continuous object deformation.

The performance of the skeletonization is again demonstrated on the liver vascular-
ization.

1.2.4 Appendices

In the Appendix, we give the code of the methods proposed in this thesis. The code is
given in an abstract metalanguage. The level of detail may correspond to implemen-
tation of the algorithms by using some specialized library (as for graph manipulation,
for example). Also, not every procedure is listed. Focus is only given to the new
methods proposed in the manuscript. None of auxiliary algorithms, such as graph ex-
traction, region labeling, is given, as it is beyond the objective of the manuscript. The
principle of the code is clarified by giving a few-word comment to every procedure.

\(^2\)while a deletion of a single simple point is a homotopic operation, deletion of several simple
points may change the topology
2.1 Objectives of the Research

In the Part II of the manuscript, we consider an existing operator which belongs originally to the mathematical morphology, the *h-minima*. To explain the principle of *h-minima*, a grey-scale image can be seen as a topographic relief where the intensity function of the image represents the altitude. A very simple segmentation of such an image can be obtained by application of the watershed partition. In real cases, the resulting segmentation, however, is often too fragmented due to the noise present in the image. This “ oversegmentation” can be alleviated by filtering the input image by using the *h-minima*. The *h-minima* operator filters out the irrelevant minima and preserves the principal basins of the intensity function. The irrelevant minima are identified by a low level of their dynamics. This segmentation method is therefore originally conceived to two-class segmentation. An indirect multi-class segmentation is possible by considering the gradient rather than the original input image. The disadvantage of this approach, which has actually motivated our work, is its inevitable loss of information caused by lower resolution of the morphological gradient.

In the manuscript we explain how the image detail is lost during the extraction of the morphological gradient. The objective of the approach studied in this part is the extension of direct applicability of *h-minima* to multi-class images, without explicit extraction of the gradient.

2.2 Principle and Experiment Results

To become independent of the geometric properties of an image, the image is represented by a weighted graph. The graph vertices represent the plateaus of the input image, i.e. the regions of a constant intensity level. For every pair of adjacent plateaus, there is an edge in the graph, connecting the corresponding vertices. The edges are weighted by the absolute intensity difference of the corresponding adjacent plateaus. This representation is freed of the geometric information contained in the image. The smallest indivisible element is therefore the vertex of the graph. The gradient is represented implicitly by the values assigned to the edges of the graph. Intuitively, the values of the edges correspond to the values of the gradient. The *h-minima* operating on the graph are no longer depending on the choice of the structuring element needed for extraction of the morphological gradient.

We show the results obtained with the modified *h-minima* operator on a photography of a corridor, see Fig. 2.1 a). The objects difficult to extract are indicated with arrows. Figure 2.1 b) gives results obtained with the original *h-minima*. The ceiling bars are segmented incorrectly. Figures 2.1 c) and d) gives results obtained with the *h-minima* modification proposed in the manuscript. The segmentation of the ceiling
Figure 2.1: Segmentation results obtained by various versions of the h-minima operator.
structure is more appropriate. The extraction of the floor tiles is also better and more precise.

2.3 Properties

Several interesting properties of the $h$-minima operator are given and discussed in the manuscript.

This new $h$-minima operator defines a partition of the image (on the contrary to the original $h$-minima, which extracts the object markers). Also, the $h$-minima as proposed in the thesis is a connected and increasing operator. Two other interesting properties of this operator are the anti-extensiveness and idempotence.

Thanks to these four properties, this operator belongs to the family of connected morphological openings. The classification among the connected morphological openings is all the more interesting that it permits the conception of more sophisticated and more powerful operators. See the result in Fig. 2.1 c), it is deteriorated with a salt-and-pepper noise which could not have been suppressed with the $h$-minima operator because the amplitude of the noise is too strong.

We give in the manuscript an example of a more sophisticated noise-filtering operator based on the surface opening, cf. the result Fig. 2.1 d). The surface opening filters efficiently the salt-and-pepper noise and preserves the objects in the image. Moreover, we have seen, that this surface opening also helps to reconnect fragmented contours.

2.4 Conclusions

We have considered an extension of an existing operator belonging to the mathematical morphology framework in the context of incidence graphs. The new operator possesses different properties and its direct applicability is extended to multi-class segmentation. The new operator is not based on the binarization obtained by successive thresholdings, which makes it different from connected openings studied in the literature (cf. [Ron], or [SS95]). It corresponds rather to segmentation according to a region homogeneity criterion which makes it applicable to the multi-class images, where the different classes are distinguished by varying light intensity.

Thanks to the increasingness and anti-extensiveness properties, letting $h$ vary from 0 to $\infty$ generates an inclusion tree of the segmentation classes. An interesting consequence is that the new $h$-minima generates a pyramid of segmentations.
3 PART III. TOPOLOGY-DRIVEN SEGMENTATION AND SKELETONIZATION

3.1 Objectives and Principles

The topology of the objects that are to be extracted is often known and determined. Nonetheless, various segmentation techniques usually don’t permit to control the topology of the result. Therefore, some post-processing technique needs to be employed to correct the topology of the segmented result. The elevated complexity of such post-processing techniques (cf. for example [Sol98]) motivates the search for such techniques that control the topology during the segmentation.

Two methods of image segmentation and one method of skeletonization are introduced in this part of the thesis. All the three methods are topologically oriented and guarantee to yield a topologically correct result. The methods are based on homotopic thinning or thickening controlled by local light intensity.

The continuous object deformation (i.e. homotopic) in a discrete space (i.e. $\mathbb{Z}^3$) is possible thanks to the notion of simple point. Simple point is such a point of the object which can be deleted without modification of the topology.

The methods use a controlled sequence of addition or deletion of simple points. The first segmentation method - $\lambda$-thickening - is based on homotopic thickening of a given marker. The extracted object is topologically equivalent to the marker. The second segmentation method - $\lambda$-thinning and the $\lambda$-skeletonization method use homotopic thinning. The initial object is the bounding box of size equivalent to the input image. If the marker given to initialize the thickening (an isolated point, for example) is topologically equivalent to the bounding box then these methods can be used to extract simply connected objects. See Fig. 3.1 explaining the principle of the $\lambda$-thinning.

The image segmentation is obtained by $\lambda$-thickening or $\lambda$-thinning operation which is stopped at the level $\lambda$ of intensity. The $\lambda$ is chosen according to the object to extract. Therefore, these methods can also be seen as topologically controlled thresholding.

3.2 Experiment Results

The performance of these methods has been tested on X-ray 3-D CT angiography scan of liver, cf. Figure 3.2 a). The input image was used raw without any preprocessing. See the results given in Figs. 3.2 b) and c).

The $\lambda$-thinning algorithm proposed in the manuscript can also be used to obtain skeletons of 3-D objects in grey-scale images. The difference between $\lambda$-thinning and $\lambda$-skeletonization consist in modification of the point selection criterion. The $\lambda$-skeletonization is also tested on a liver vascular system which adopts a tubular form with rich ramifications. This structure is best represented with the curved-line skeleton.
3.3 Properties and Conclusions

Nonnegligible and interesting property of these methods is a very low (almost linear) calculation complexity. Linear calculation complexity is very advantageous when 3-D images are processed because of the considerable data volume.

The $\lambda$-thinning and $\lambda$-thickening methods are dual. They are, however, not complementary in the sense that the part of background deleted by the $\lambda$-thinning is not complementary to the object obtained by reconstruction by $\lambda$-thickening of the marker.

4 PERSPECTIVES

4.1 Part Graphs and Image Segmentation

Future research concerning the improvement of the segmentation quality seems promising, as the conception of more complex operators from the family of connected open-
Figure 3.2: Results of segmentation and skeletonization.
ings. At this moment, two subjects of research concerning the improvement of the results seem possible:

Undersegmented, disconnected contours could perhaps be reconnected by employing other criteria as geometric form or presence in such an area of the input image which is characterized by locally strong dynamics. These criteria could be used to enable fusion of resembling regions even if their intensity difference exceeds \( h \). Such a criterion would reflect geometric resemblance while permitting to keep a lower global value of \( h \).

Also, locally adaptive value of \( h \) rather than a global value applied to the entire image would help to improve the quality of segmentation of such images that are characterized by spatially varying dynamics.

### 4.2 Part Topology-Driven Segmentation and Skeletonization

The necessity of experimental choice of good parameter value is inconvenient. The value of the parameter may however be adjusted during the process of homotopic reconstruction. As soon as the object has a satisfactory form the process can manually be stopped, or the reconstruction can be continued.

A statistic-based method is considered in the manuscript to estimate a good value of the \( \lambda \) criterion. This method is based on a histogram analysis.

The methods proposed in this part of the manuscript are originally conceived for simply connected objects. Extraction of objects containing several connected components is however possible by using the \( \lambda \)-thickening. If the \( \lambda \)-thickening is initialized with several markers, each for one connected component of the object, then the result of the reconstruction will be topologically equivalent to the set of the markers.

All the methods derived from this approach are based on the notion of simple point. The reconstruction proceeds in a sequential way - one simple point in every iteration. A parallel reconstruction could also be conceived by using the notion of \( P \)-simple point introduced by G. Bertrand.


**Curriculum vitae**

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**Publications**


P. Dokládal and G. Bertrand: “Application of $h$-minima to multi-class images using graphs”, Conference proceedings, Graph based Representation IAPR-15, Sankt Pölten, Austria


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